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Evacuation modelling analysis within the operational research context: A combined approach for improving enclosure designs

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ABSTRACT

Evacuation models have been playing an important function in the transition process from prescriptive fire safety codes to performance-based ones over the last three decades. In fact, such models became also useful tools in different tasks within fire safety engineering field, such as fire risks assessment and fire investigation. However, there are some difficulties in this process when using these models. For instance, during the evacuation modelling analysis, a common problem faced by fire safety engineers concerns the number of simulations which needs to be performed. In other terms, which fire designs (i.e., scenarios) should be investigated using the evacuation models? This type of question becomes more complex when specific issues such as the optimal positioning of exits within an arbitrarily structure needs to be addressed. Therefore, this paper presents a methodology which combines the use of evacuation models with numerical techniques used in the operational research field, such as Design of Experiments (DoE), Response Surface Models (RSM) and the numerical optimisation techniques. The methodology here presented is restricted to evacuation modelling analysis, nevertheless this same concept can be extended to fire modelling analysis.

1. Introduction – evacuation modelling and fire safety codes

The first aim of fire safety is to provide the occupants' safety in enclosed environments, avoiding and/or reducing the number of fatalities (and/or the number of injuries) [1]. Nevertheless, this task is challenging considering the actual growing complexity of the architectural designs, which introduces more fire risks. In order to address this issue, the fire safety codes have been changing from a prescriptive approach to a more performance-based one over the last thirty years [1]. Some countries like the U.S.A., Canada, Sweden, New Zealand, Australia and the United Kingdom are in an advanced stage of development and implementation of the performance-based codes. Evacuation and fire models have been developed to enable this process.

In fact, the evacuation and fire models have been playing an important function in this process, since they help to assure that the solutions proposed by performance-based codes are feasible and are able to address fire safety issues correctly. Bryan [2] says that this "worldwide movement towards performance-based codes has created a demand for computer evacuation models that will provide an estimate of the evacuation time for a building". S. Ko et al. [3], says that "fire engineers often use evacuation models to assess buildings and their ability to provide sufficient time for the occupants to evacuate safely in the event of a fire or other emergency". Galea [4] also re-enforces it when he says that "the complex demands on design spaces challenge traditional prescriptive design guides and regulations. Designers and regulators are consequently turning to performance-based analysis and regulations facilitated by the new generation of people movement models" (i.e., evacuation models). In reality, the development of such models became an important field of research and work within the Fire Safety Engineering (FSE) industry. This field is commonly denominated by specialists as Computational Fire Engineering (CFE), see Fig. 1.

In Fig. 1, it is possible to see that the FSE community is responsible to develop the fire safety codes. These fire safety codes, as mentioned previously, have been changing from a prescriptive approach to performance-based one. And the CFE models are being used to help in this process. For instance, the BS7974 (British Standard) is an example of it, once this fire safety code follows a performance-based approach and the use of evacuation models has been enabling its validation [5]. Within this context, particularly, the evacuation modelling became a popular area of research within the FSE community.

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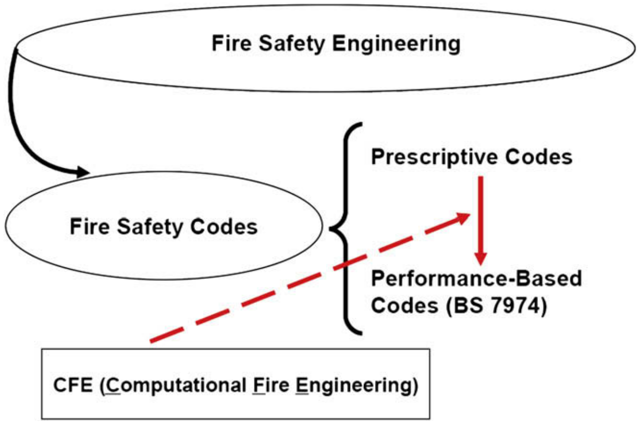

Fig. 1. The use of CFE models to address fire safety issues [5].

In reality, for the last few decades, as mentioned before, the evacuation models have been used to address fire safety issues within complex structures, where the prescriptive codes, generally, do not provide clear guidance. For this reason, these models have been largely applied for estimating the RSET (Required Safe Egress Time), instead of the use of hand calculations approach. Fahy [6], also agrees with this statement, when she says that evacuation models are important tools for the evaluation of engineered designs, because such evaluations require the estimate of safe egress time for the occupants.

In other terms, it could be said that there are essentially two methods available for calculating evacuation times, the more traditional hand calculation approach and with the use of evacuation models. The estimation of the evacuation times using the hand calculation approach often follows the equations provided in the Society of Fire Protection Engineers (SFPE) Handbook [7]. Although it is possible to get a good indication of the total evacuation times in relatively low populated enclosed environments by using the hand calculation approach, the introduction of significant areas of congestion in highly populated buildings and structures means that a more appropriate method of calculation is to use one of the many evacuation models available. Therefore, evacuation models became useful tools within the FSE community.

Furthermore, evacuation models have been developed largely over the last few decades. They are being used in a wide field of applications, such as crowd dynamics in open spaces, pedestrian movement in assemblies, human behaviour in evacuation process (i.e., commonly called also as egress process) during emergency situations in enclosed environments, etc. (and beyond the FSE community, evacuation models have been the object of study in many other fields of knowledge such as Risks Assessment/Safety Sciences, Crowd Management, Operation Research, Artificial Intelligence/Computer Modelling, and many others [5]). Therefore, evacuation models became important sources for the understanding of evacuation processes in general.

Nowadays, there are over 40 evacuation models. They can be used for different types of enclosed environments, such as: buildings, aircraft, ships and trains. For instance, Pelechano and Malkawi present an interesting work discussing the use of evacuation models for simulation evacuation processes in high rise building [9]. All of these models do have their advantages and disadvantages. But, in general terms, what makes them different from each other is the way they represent the geometry of the structure, the occupant's characteristics, etc. And besides that, the manner that their inherent algorithms work, will determine how accurate the evacuation model is. In the literature, there are some few evacuation models' reviews. Friedman [8] can be mentioned as the "pioneer" of such kind of reviews. Olenick and Carpenter [10] have updated this survey. Their work is internationally well known and available. Therefore, it is not the objective of this paper to analyze in depth evacuation models.

In the next section, the concepts of safe design in terms of evacuation processes efficiency are discussed.

2. Evacuation processes and safe design

First of all, it is important to define the relation between evacuation processes and safe designs (specifically, in terms of fire safety). These two concepts are essentially linked to each other. Evacuation Process could be simply stated as the escape movement that the occupant(s) of an enclosure makes under emergency situations, such as fires, earthquakes, flooding, explosions, terrorist attacks and so on. And the safe design, given the enclosure environment's configuration, is the design which could provide a successful evacuation process (i.e., no injuries and no deaths) of its occupants in case of an emergency situation [1], as defined previously. In terms of fire safety, the emergency situation is the fire. As mentioned previously, fire safety can be measured in terms of number of fatalities, therefore in a safe design, the probability of a successful evacuation process (i.e., no fatalities) is very high.

In this specific case where the emergency situation is a fire, the safe design is commonly established numerically by following inequality [7]:

$$\text{RSET} < \text{ASET} \qquad (1)$$

where RSET means the Required Safe Egress Time; ASET means the Available Safe Egress Time.

Fig. 2 presents a set of timeliness which help the understanding of this inequality.

From Fig. 2, these are the meanings: IG – Ignition (the point when the fire starts); DET – Detection (the point when the detection systems are activated; i.e., sprinklers and etc.); AL – Alarm (the point when the alarm is sounded); REC – Recognition (the point when the occupants recognize that an emergency situation is taking place); RESP – Response (the point when the occupants respond to the situation for starting the escape movement); EVAC – Evacuation (the point when the occupants start to evacuate); UC – Untenable Conditions (the point when the fire products, i.e., smoke, heat, toxic gases, narcotic gases, irritant gases etc. kill the occupants).

The pre-movement time is also known as pre-evacuation time. The difference between the ASET and the RSET is what the FSE community calls as the "safety margin". Therefore, Fig. 2 shows clearly what is needed for a successful evacuation process, where the ASET is bigger than the RSET.

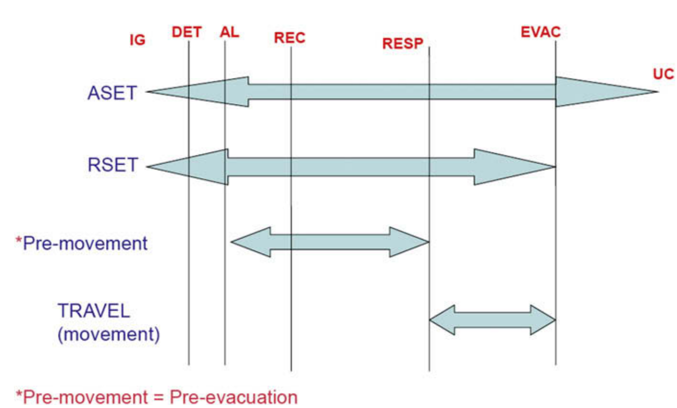

Fig. 2. The important timeliness during an evacuation process.

In practical terms, the RSET should be within the *pre-flashover* period. The Fig. 3 illustrates this.

Therefore, the occupants should be able to escape before the FRI point is reached, when the flashover occurs and the untenable conditions are already reached.

This understanding surely can be extended to other types of emergency situations as mentioned previously. Furthermore, the ASET would be substituted for the *timeline* associated with the correspondent emergency situations and then compared with the RSET. For instance, if instead of a fire, an earthquake takes place near to the analyzed enclosure, then the time that the enclosure's structure would take until collapses should be estimated and compared with the RSET and so on.

In summary, the following statement can be done:

Safe design = Successful evacuation process = No fatalities

In the next section, the process of how the safe design is defined are presented and discussed.

3. Defining safe design process – DSDP

As discussed previously, the CFE (Computational Fire Engineering) models are used to estimate these two main timeliness: RSET and ASET. The evacuation models are used for estimating the RSET and the fire models, for estimating the ASET. (In more complex analyses, the combination of these two models should be used, once the fire products affect the occupants' movement and decision-making behaviours and consequently the RSET.)

Therefore, to assure that a given design could provide this condition between these two timeliness, the designers must attempt to consider a set of aspects (which here are going to be called as criteria), including, for instance the safety of the occupants. In reality, the classical design concept usually attempts to satisfy some basic criteria, such as: comfort, functionality, maintenance, cost/benefits and aesthetics. However, when defining the safe design, another criterion should be considered as well, namely: the safety of the occupants.

In practice, these criteria can be conflicting and to develop a safe design is a challenging task for designers [1,27]. Based on this, the challenge is to combine these criteria satisfactorily in a way that the safe design can be achieved. For this task, the first set of main questions is: "how can we develop a safe design satisfying these criteria, given their conflicting nature?" And, "how can we manage to deal with the real constraints?" The development of a safe design can be understood as a "*multicriteria* decision-making problem", once more than one criterion must be taken into account as Fig. 4 shows.

From Fig. 4, it is possible to observe that during the development of the safe design, several "designs" (i.e., scenarios) are analyzed, considering the criteria, mentioned previously.

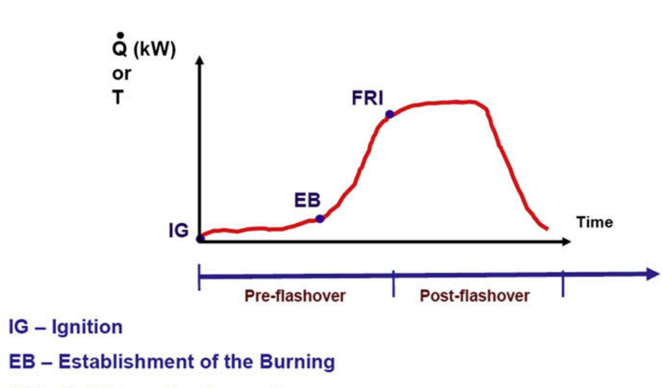

Fig. 3. Typical curve for a fire in an enclosure (adaptation from Kawagoe [11]).

This process can take a considerable amount of time, depending on the case. And as mentioned before, this is also why the CFE models, particularly, the evacuation models have been used to help the designers to develop a safe design accurately and faster.

Nevertheless, even with the use of CFE models, this design process can be still time consuming. And depending on the complexity of the design this could take too long and consequently a substantial amount of money, which sometimes, in practical life, is not possible to be spent. Figs. 5 and 6 show what here in this paper is being called as the Defining Safe Design Process (DSDP).

From Fig. 5, it is possible to observe that the evacuation modelling process, depending on the case, can take too long in order to define the safe design. And Fig. 6 presents this in a more detailed structure.

Putting this into practice, the first block event represents the design defined by the architectonic plan. At this stage, it is unclear if the design would provide or not a successful evacuation process for its occupants in case of emergency situations. (This process can be also applied to existent designs.) Scenario(s) then is (are) defined in order to check and/or define the safe design. At this stage, the use of evacuation models takes place. The number of scenarios which will need to be analyzed and simulated will vary from case to case.

From this perspective, it is possible to understand that in some cases, this process will be straight forward, however in other cases, this might not be possible. Indeed, there are some additional questions which are implicit and might be done (these questions are inserted into the second main set of questions): "is it possible to reduce the number of simulations?"; "what can we do in order to reduce the number of simulations, without compromising the safety?" and "for how long should we run the simulations to assure this?"

These kinds of questions are likely to occur during the DSDP and the designers will have to deal with them. Based on that, this study presents a new approach which aims to help answering these questions stated before:

The first set of questions:
How can we develop a safe design satisfying these criteria, given their conflicting nature?
How can we manage to deal with the real constraints?
And the second set of questions:
Is it possible to reduce the numbers of simulations?
What can we do in order to reduce the number of simulations, without compromising the safety?
Until when should we run the simulations to assure this?

Furthermore, in reality, the question which could cover these two set of questions is: "how can we optimise the DSDP"?

First of all, a good understanding of the manner in which the core variables interact to control the evacuation efficiency is an important issue to be addressed and consequently to bring some light to it.

In reality, given that time is the basic measure of the evacuation process [12], the evacuation time can be taken then as an index of how successful the evacuation process would be in an enclosure. Following this, it can be assumed that the lowest the evacuation time is, the safest the design becomes. In the next section, this issue is discussed in details.

4. The evacuation time

The evacuation time, according to what was mentioned before, can be seen as an index of the efficiency of the evacuation process. In other terms, the evacuation time is the variable used to measure the evacuation process' performance. In fact, when analyzed

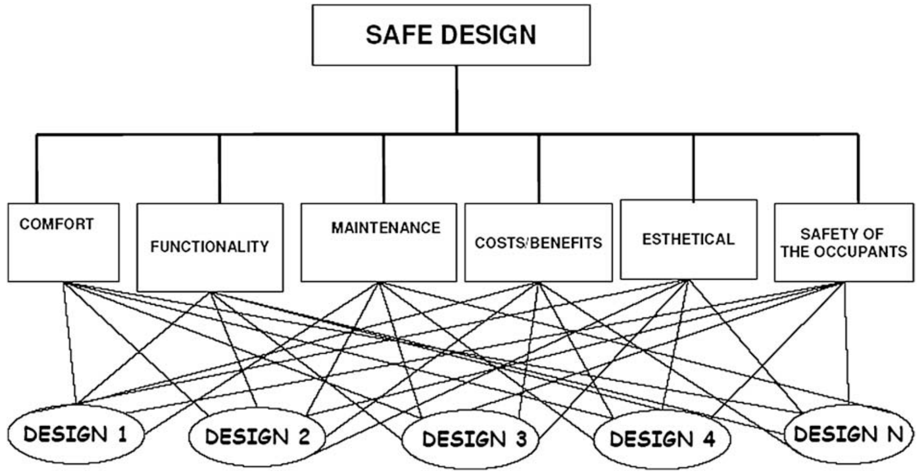

Fig. 4. Defining a safe design taking into account the criteria [1].

deeply, the evacuation time brings also the idea of key-aspects in the evacuation process' context, such as congestion (C) and flow rate (FR) [5].

In a safe design, when an evacuation process takes place, the ideal situation is found: there will be few congestions (and in some case, there will be no congestions) which enable a good movement of its occupants (i.e., the flow rate is high). And as consequence of this, the evacuation time is reduced. Therefore, the evacuation time is an important index of how efficient the evacuation process is within a specific design, in other terms, it can indicate how safe the design is. The ideal condition is:

$$C\downarrow \; FR\uparrow \; ET\downarrow$$
$$\text{(RSET < ASET)}$$

Furthermore, this condition, in technical terms, should be the main objective of fire safety engineers when doing the evacuation analysis: to reduce the congested areas for improving the flow rate and consequently allowing the occupants to evacuate faster and safely.

Based on that, this study took the evacuation time as the main reference to analyze the evacuation process.

In the literature, many authors have suggested formulas for the calculation of the evacuation time. For instance, Buchanan [13] gives the following equation:

$$t_{ev} = t_d + t_a + t_o + t_i + t_t + t_q \quad (2)$$

where t_{ev} is the time to evacuate (i.e., evacuation time); t_d is the time from the ignition point until detection of the fire; t_a is the time from detection until the alarm is sounded; t_o is the time from alarm until the time occupants make a decision to respond; t_i is the time for the occupants to investigate the fire, collect belongings, fight the fire, etc.; t_t is the travel time or the movement time, (which is the actual time required to escape route until a safe place, like an assembly point, including way-finding); t_q is the queuing time at corridors, exits and/or other places/obstacles in the enclosure.

Sime [14] summarizes the evacuation time as being a simple equation as follows:

$$\text{ET} = t_1 + t_2 \quad (3)$$

where ET is the evacuation time; t_1 is the time to start the movement; t_2 is the time to move and pass through the exits.

The first time (i.e., the time to start the movement) is also commonly called as pre-evacuation time (i.e., pre-movement time) [15]. In fact, it is a variable which depends on the psychological attributes of the occupants. These attributes have been researched by many specialists in the FSE community dedicated to address human behaviour in fires. This issue is an important and complex aspect of the evacuation process. Nevertheless, this study is not dedicated to cover the time to start the movement, as mentioned previously. Instead, the focus of this study is the second time (i.e., the time to move and pass through the exits). Therefore, this study intended to investigate some important factors which impact this time. Based on that, similarly to the equation proposed by Sime [14], another equation can be rewrite:

$$\text{ET} = t_M + t_E \quad (4)$$

where ET is the evacuation time; t_M is the time spent during the movement; t_E is the time spent towards the exits.

Clearly, in this equation, just the interaction occupants–structure is being considered. The interaction occupants–occupants is also being considered, but only in terms of the physical aspects (i.e., in terms of how the occupants interact to each other during the movement).

The first time (i.e., the time spent during the movement) is influenced mainly by: the lay-out of the enclosure, the travel distance, the number of occupants and the enclosure geometry's features.

The second time (i.e., the time spent towards the exits) is influenced mainly by: the exit location, exit width and the number of occupants.

In reality, one of the issues which make the DSDP complex is the determination of the optimal positioning of exits around the

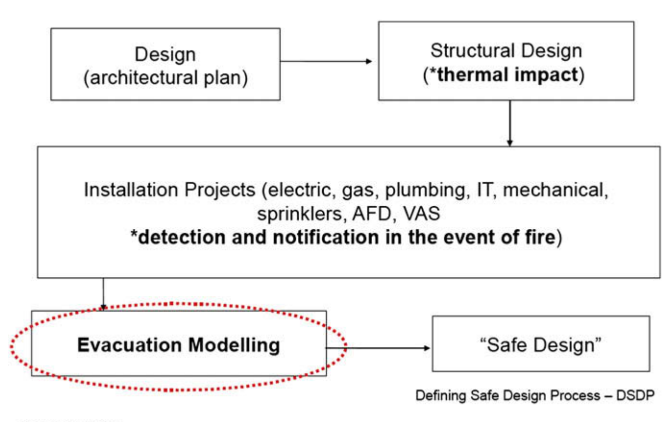

Fig. 5. Defining safe design process – DSDP.

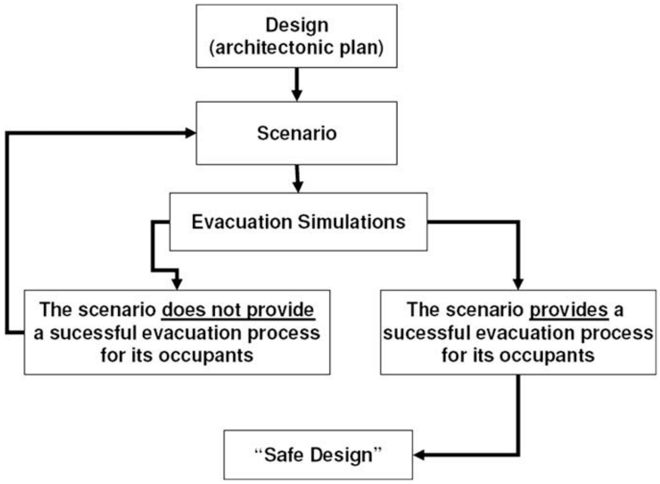

Fig. 6. Defining safe design process – DSDP.

perimeter of the design geometry. The solution is found through trial and error exploration of the possible significant exit locations. Similarly to the existent problem in the fire modelling field in terms of sizing and locating vents extractors, this is also a common problem faced by fire safety engineers in evacuation analysis: the optimal positioning of exits within an arbitrarily complex structure in order to minimize evacuation times. To a certain extent, building codes provide guidelines for the positioning of exits, however this is not very clear in terms of how to minimize the evacuation time. For example, given an arbitrarily complex room, ignoring constraints imposed by regulations such as minimizing travel distances and avoiding dead-end corridors, where should exits be placed in order to minimize evacuation times? Indeed, for an arbitrarily shaped room with a given number of exits, does the distribution of exits around the perimeter impact the evacuation time?

This problem becomes more difficult as the available options and hence complexity of the evacuation scenario increases. It can reasonably be expected that for a given population size, the solution of the problem will be dependent on the shape and size of the compartment, the number and relative size of the available exits. For a specified problem, the engineer could examine several possible exit location options and select the configuration which produces the smallest evacuation time, but this would not necessarily produce the optimal configuration or the global minimum evacuation time. Using this approach, the engineer would have to examine every significant combination of exit location to be sure that the global minimum had been found. For an arbitrarily complex shaped room with a large number of exits of varying size, the number of possible permutations of exit size and location would measure in the hundreds if not thousands.

Therefore, the question now is: how would the engineer find the best solution and how would the engineer know that an optimal or near optimal solution had been found? A possible answer to this problem may be found in the operational research field through optimisation theory.

In reality, the numerical optimisation techniques have been applied in a range of different fields such as structural analysis and have been shown to be powerful tools for designers, saving time and reducing costs. The use of classical optimisation theory concept and its associated fields (such as Design of Experiments, DoE, and Response Surface Models, RSM) are here explored and inserted in evacuation simulation analysis.

Therefore, this is one the main objective of this study, to present and discuss a systematic methodology to efficiently optimise evacuation safety aspects of structural designs. Such an approach will be of particular interest to practical fire engineers as it allows the fire engineer rapidly and efficiently optimise their design.

In the next section, the optimisation theory is discussed briefly.

5. Optimisation theory

There are a vast field of activities/tasks in the everyday world which can usually be described as systems; from actual physical systems such as chemical power plants to theoretical entities, like economic models [16]. According to Campello [17], systems, in general terms, are a collection of objects connected through any form of interaction or interdependence. The efficiency of these "systems" often requires an attempt at the optimisation of a set of indices which measure the performance (i.e., behaviour) of the system. These indices, when quantifiable, are represented by algebraic variables. Then, values for these variables must be found which maximize the gain or profit and minimize the waste or loss of the system. Foulds [16] says that this process of maximization and/or minimization of the system is known as optimisation. In other terms, optimisation is the process by which the optimal, or optimum, solution to a problem is produced. (The word optimum has come from the Latin word "*optimus*", which means best.) Finding the optimal solution of a certain problem, based on the analyzed system features, follows generically a methodology, which involves several tasks [18], see Fig. 7.

In general terms, the optimisation problem involves an objective function (i.e., the dependent variable, response, output, merit function) which is needed to be minimized or maximized. The general formulation of a classical optimisation problem is as follows:

Maximize or minimize:

$$\mathrm{OBJ}(X, Y, Z \ldots n)$$

Subjected to:

$$g_j(X) = 0 \quad (j = 1, m)$$
$$h_k(X) \leq 0 \quad (k = 1, L)$$
$$X^L \leq X \leq X^U$$

where the objective function is $\mathrm{OBJ}(X, Y, Z \ldots n)$. The equalities constraints are $g_j(X) = 0$. The inequalities constraints are $h_k(X) \leq 0$. The number of equalities constraints m. The number of inequalities constraints L. The Lower and Upper bounds of the design variables areX^L and X^U.

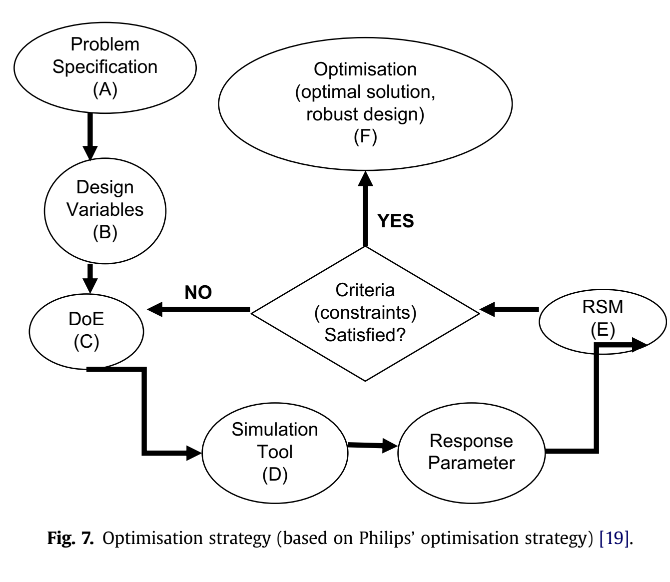

Fig. 7. Optimisation strategy (based on Philips' optimisation strategy) [19].

This study proposes a methodology in which evacuation processes are analyzed under this perspective. The objective function is the evacuation time and the problem is to minimize this function.

Fig. 7 presents the methodology adopted in this study.

In this particular study, this methodology follows a set of steps in which involve, amongst other computational packages, the use of computer simulation software to simulate the evacuation process.

Some of these steps, such as, the problem specification (step A) and the design variables (step B) are described in the following paragraphs of this section. The simulation tool (step D) is the evacuation models, discussed previously. And regarding the Design of Experiments Techniques (i.e., DoE techniques, step C), Response Surface Models (i.e., RSM, step E) and optimisation (step F), these steps are discussed in the next sections of this paper.

Furthermore, an overview of the classical optimisation problem is, first of all, to define the design variables (i.e., factors) which impact the performance of the system. Once they are defined, data must be produced in order to analyze their relations amongst themselves and the consequent impact generated from these relations on the system. (The data are produced from laboratory experiments and/or computational simulation packages. In this study, the Building EXODUS evacuation model [20–23] was used for generating data.) For this, the obtained data, which represent numerical values for the design variables, are organized in a *design space* through a response surface. Therefore, a response surface model is then developed with the purpose to describe this response surface. And finally, given that the *response surface* is described by a function (which composes the objective function), numerical optimisation techniques are applied to optimise this function (i.e., maximize and/or minimize). (All of these concepts of design space, response surface etc. are also discussed briefly in the next sections of this paper.)

Therefore, in summary, the optimisation strategy follows this basic sequence: i) use of DoE techniques for generating data; ii) use of RSM for generating the response surfaces, enabling later data analysis; iii) use of numerical optimisation techniques for solving the problem. Fig. 8 presents this methodology in a systematic approach.

In this study, evacuation processes are analyzed under this perspective through the use of evacuation models.

In the next paragraphs, this is discussed further.

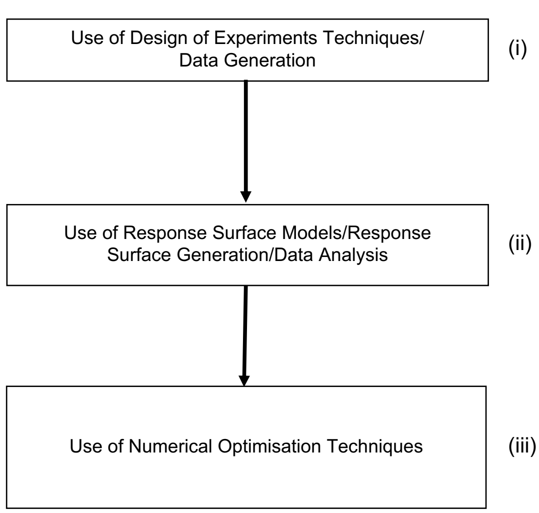

Fig. 8. Summary of the optimisation strategy used in this study.

5.1. *The problem specification: classical optimisation problem "versus" evacuation processes*

The problem specification (step A from Fig. 7) is defined here. For this, the following questions should be addressed, namely:

What is the system?
Which variable will be taken as the objective function (i.e., to represent quantitatively the system)?
Which variable(s) will be taken as the design variable(s) of the function?
Is the problem a minimization problem or a maximization problem?
Is the problem a constrained or an unconstrained problem?

According to what was mentioned previously, optimisation problems involve an objective function which is needed to be minimized or maximized. In this study, the objective function is the evacuation time and the problem is to minimize it.

Regarding the design variables (i.e., independent variables, factors, inputs), once there are several variables which do have direct and/or indirect impact on the evacuation process performance, they can be: exit(s) width, exit(s) locations, number of exits, relative distance between two exits, number of occupants, response times, shape of the room, type of fuel package within the enclosure, location of the fuel package, lay-out of the enclosure, etc.

Based on that, the questions are then answered.

For this study, the system is the evacuation process.

As mentioned previously, for this study, the evacuation time is taken as the objective function.

The design variables could be the exit(s) locations along the perimeter wall, relative distance between the exits etc.

Also, according to what was mentioned before, the problem here is a minimization problem, given it is assumed that the lower the evacuation time is, safer the design becomes.

And regarding the nature of the problem (i.e., if the problem has constraints or not), in this study, both problems can be considered depending on the nature of the design variables: with constraints and with no constraints. This is discussed further in this section.

In Table 1, the concepts found in the optimisation theory were associated with the concepts used in the FSE field.

Therefore, given their nature, evacuation processes are complex systems in nature. According to Capra [24], a complex system is any system which has more than one variable influencing its behaviour along time. In fact, when the variables (which might influence and those which surely influence the evacuation process' development along a period of time) are observed more carefully, this concept can be brought into evacuation analysis.

For instance, as mentioned before, there are several variables which do have direct and/or indirect impact on the evacuation process performance.

For this reason, it is correct to say that the evacuation time can be seen as a multivariable function, once it describes the behaviour of a complex system: the evacuation process. Based on that, mathematically speaking, the problem analyzed here is described as follows:

Evacuation Time (ET) = objective function and can be written as:

Table 1
Summary of the problem.

Optimisation theory concepts	Fire safety engineering concepts
System	Evacuation process
Objective function	Evacuation time
Design variable(s)	Exit(s) locations; number of people etc.

$$ET = f(x, y, z \ldots n)$$

where x, y, $z \ldots n$ represent the design variables.

Following this thinking, these variables are interconnected to each other and influence the evacuation process, and this influence is measured by the evacuation time. The understanding of these variables and their relationship amongst each other as well as the consequent impact of them and their relations on the evacuation time is an important issue to addressed [25]. Fig. 9 illustrates this.

As mentioned before, this study intends to present an analytical methodology which can provide an optimised analysis of designs in terms of fire safety of the occupants. This methodological approach combines the use of concepts of optimisation with the use of evacuation modelling analysis. At the same time, with the implementation of this methodology, it is also expected that the manner in which the core variables interact to control evacuation efficiency should be investigated.

From Fig. 9, it is possible to see that the design variables influence the evacuation process and this influence can be measured by the evacuation time. In fact, Fig. 9 can be visualized differently using a simple block diagram in where the steps shown in Fig. 7 can also be associated, as Fig. 10 presents.

In the next paragraphs, the design variables are discussed.

5.2. The design variables

The design variables (step B from Fig. 7) are here discussed. As mentioned before, in addition to investigating the use of optimisation theory for evacuation applications, this study is also interested in investigating the fundamental relations between design variables and evacuation, i.e., amongst the design variables themselves and between them and the evacuation process. This is also a relevant aspect of this study.

It might be relevant to observe that these design variables come from the interaction occupants–structure. In addition, it is important to note that, the evacuation time is not a design variable, because it is a dependent variable, given that its value will depend on the values and interaction between all of the design variables. And in the other hand, the design variables were found to be independent of one another, i.e., their values do not impact each other's value. For instance, if the width of the exit is increased, it will not affect the exit location and vice-versa. This is a fundamental concept of design variable. Therefore, it is very important indeed to have a good understanding of the nature and the relationship between design variables in order to construct robustly the problem. Otherwise, if this understanding is not clear, there is the risk of variables which are not design variables be mistaken as so and this is crucial for the whole optimisation process.

According to what was explained previously, there are many design variables which influence the evacuation efficiency.

Three design variables which could be mentioned (given that they influence directly and/or indirectly influence the evacuation process) are: exit width, exit location and relative distance between exits.

The relation between evacuation time and exit width is already well known. The wider the exit is, the lowest values the evacuation time assume. The reason for that is based on the fact that when the exit becomes wider, the flux of people passing through the exit (i.e., the flow rate) becomes higher and consequently evacuation process can be completed in shorter times. This relation is logical and already well known.

Nevertheless, considering that the fire safety designer would like to analyze this relation based on the methodology here proposed. He/she would then need to define the domain of this variable. Assuming that he/she wants to check the evacuation times for a specific enclosure for the exit width varying from 1.0 m to 2.5 m. Therefore, this is the domain for the exit width (EW).

If the exit width can assume all the possible values from 1.0 m to 2.5 m then, this is a continuum variable. Otherwise, it would be a discrete variable.

Therefore, in general terms, the problem based on this relation between exit width and evacuation time would be stated as follows:

$$\text{Minimize}: \ ET = f(EW), \quad \text{where } 1.0 < EW < 2.5$$

Regarding the relations between evacuation time and exit(s) locations and also between evacuation time and the relative distance between exits are not fully understood. For instance, there is no clear guidance regarding:

Where to place an exit in order to produce minimum evacuation times?
Is it better to have two exits of X m or one exit of $2X$ m?
If we have two exits, what is the optimal relative positioning of these exits?

The investigation of these fundamental questions is not a simple task. Nevertheless, defining the optimal positioning of exits within an arbitrarily complex structure is one of the key-issues within evacuation modelling analysis.

In fact, the exit location constitutes an important aspect to look at in terms of evacuation efficiency, because it impacts substantially the evacuation time [25]. For instance, it was found that depending on where the exits are located, the evacuation time might increase or decrease substantially [25]. These issues, namely the relations between evacuation times-exit(s) locations and evacuation times-relative distance between exits, have been investigated by the author [25]. Therefore, this is not the focus of this paper.

The exit location (EL) can be seen as a continuum variable. The exit location can move from the corner of the room until the middle of the wall. The EL values can vary from 0 (i.e., the lower value) to some maximum value (i.e., the upper value) which represents the other extreme of the perimeter of the enclosure's geometry. Fig. 11 illustrates this.

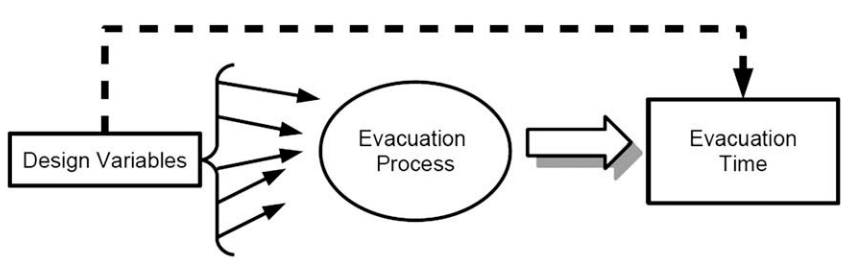

Fig. 9. The design variables influence on the evacuation process.

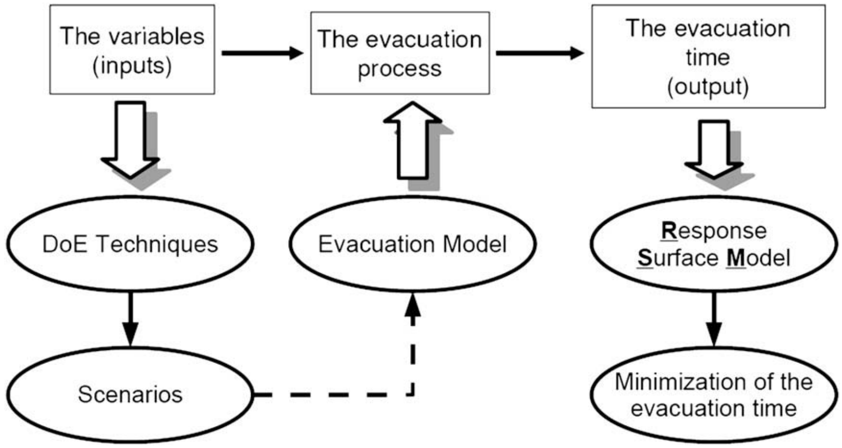

Fig. 10. Block diagram associated with specific tasks from the optimisation process.

In Fig. 11, it is possible to see that the perimeter of the enclosure is 30 m. But assuming that the exit width is 1 m, then the upper bound is assumed to be 29 rather than 30. Therefore, the exit location can be placed anywhere along this domain (between 0 and 29). The number 0 (which represents the lower bound) is representing the starting point (i.e., any corner along the wall). The domain is graphically represented by the line. Assuming that the orientation is anti-clockwise and also that the distance from the exit to a starting point is measured from the left edge of the exit to the starting point, then the EL values could be for instance:

0 – This means that the exit is located in the corner (the starting point);
1 – This means that the exit, from its left edge, is located 1 m far from the corner;
1.5 – This means that the exit, from its left edge, is located 1.5 m far from the corner;
3 – This means that the exit, from its left edge, is located 3 m far from the corner.

Furthermore, in general terms, the problem based on this relation between exit location and evacuation time based on this simple hypothetical example would be stated as follows:

$$\text{Minimize}: \ \text{ET} = f(\text{EL}) \quad \text{where } 0 < \text{EL} < 29$$

In the next section, a basic review on Design of Experiments techniques, Response Surface modelling and the numerical optimisation techniques is presented.

6. Brief review on DoE techniques, RSM and numerical optimisation techniques

6.1. The design of experiments (DoE) techniques

First of all, it is important to define what an experiment is. In practical life, designers (or investigators) perform experiments in order to discover something about a particular process or system. Therefore, an experiment can be defined as a test or series of tests in which purposeful changes are made to the input variables of a process or system so that it is possible to observe and identify the reasons for changes that may be observed in the output response [26].

Thus, when an experiment, or a set of experiments, is defined (i.e., designed) a methodological process is then established. This "process" is named design of experiments, commonly known as DoE. The problem of experimental design or design of experiments (DoE) is encountered in many fields. The common situation is when the designer does not know the exact underlying relationship between responses and design variables (of a specific system), but wants to know how the responses are influenced by these design variables [27]. (In our specific case, we want to know how the exit positions impact the evacuation process, this impact is going to be measured by the evacuation time.) In this case, it is often helpful to approximate the underlying relationship with an empirical model, which is called as response surface model (RSM) or curve fit [28], (this is discussed in further detail in topic E of this section). To create the RSM, it is needed to know the value of the responses for some combinations of design variables. Each combination of design variables could be viewed as a point in the *n*dimensional *design space*, where *n* is the total number of design variables [29]. The particular arrangement of points in the design space is known as an experimental design or design of experiments (DoE) [30,31].

So, in summary, the main purpose of DoE is to help the designer to create an experimental design, analyze the characteristics of this design, create the response surface model for this design, and later on analyze the characteristics of the response surface model.

Nowadays, there are a large number of DoE techniques. In fact, these techniques are based on logical algorithms which define the nature of the combinations between the design variables (i.e., *design points*), in other terms, the number of design points needed and the location of these design points in the design space. Basically, the difference between the DoE techniques is based on how their algorithms work.

In this study, the Latin Hypercube and the Central Composite Design (CCD) were the DoE techniques used.

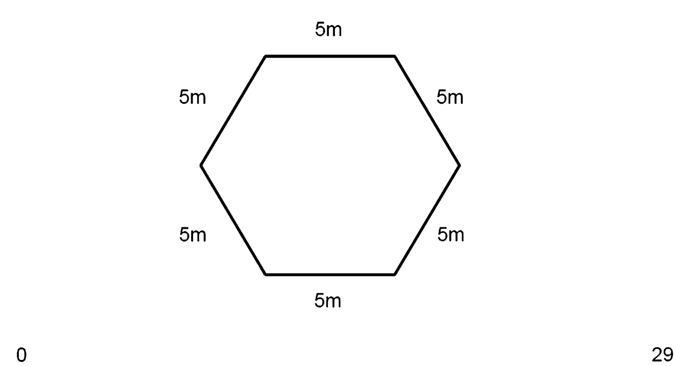

Fig. 11. Representation of the domain for the exit location in a hypothetical enclosure.

6.1.1. Latin hypercube

This technique is much known. It is a reasonable good technique indeed. It uses as a common rule a uniform probability distribution to define its special design (i.e., to picks the design points). Therefore, this technique is based on statistical criterion, which might provide some consistence for the generated results. The algorithm in this case, for instance, works like this: the set of values obtained for the first design variable are combined randomly (but equally likely combinations) with the set of values of the second design variable. These pairs are combined again with the set of values of the third design variable and so on, until to construct kind of a matrix.

6.1.2. Central composite design (CCD)

This technique, as the name suggests, picks up the design points which are located in the central of the edges of the design space. There is an advantage of using this technique which is that this technique just requires a small number of design points, given that in some cases to pick up a big number of design points could be expensive. The other aspect, which was mentioned before, and which could be seen as an advantage is the fact that this technique picks the points which are located in the centre of the edge of the design space, so with this, we can cover important locations/regions of the design space.

6.2. Response surface models (RSM)

The use of RSM has become a popular tool for multi-disciplinary optimisation [28,29]. The RSM use mathematical models to approximate the objective functions of a system in the design space [28]. Subsequently, the optimum search is performed on the response surfaces [31]. There have been many different RSM proposed, including polynomials, *adaptive splines*, radical basis functions etc [31]. For this reason, the use of the RSM constitutes a very important task for the whole optimisation problem. The reason for that is because, depending on how the curve fit was defined, the objective function is going to be represented by this fitting. So, even that the numerical optimisation technique was selected accurately, if the RSM was not well defined, the final optimum solution might not represent the real best solution.

In other terms, if the RSM chosen to fit the curve does not cover properly the design space of the problem, when the numerical optimisation technique is then applied, the result found from its use may not be realistic. In other terms, if it is a minimization problem (which is our case), for instance, the search algorithm from the numerical optimisation technique would find the local minima region instead of the global minima, and in some cases, it might even found a region completely far from the optimum solution (see Fig. 12). In summary, the RSM is responsible to “arrange” all the design points previously generated and this arrangement is crucial for solving the problem.

The curve A represents the real response curve and the curve B represents the curve fitting proposed by the RSM. We can see clearly that for this hypothetical example, the chosen RSM is not appropriate. The dark dots, which represent the global minima regions for both curves, are far from each other. If we have applied the numerical optimisation technique for this case, we would found a negative solution which does not correspond to the real solution which is positive.

Therefore, once the objective function is built and known based on the RSM, in case of being a constrained problem, it should be checked if the established constraints are satisfied. If so, then it is possible to use the numerical optimisation techniques to solve the problem. Otherwise, i.e., in case of the constraints are not satisfied and/or the curve fitting does not represent properly the design space in question, it will be necessary to change the DoE technique in order to rebuild the design space and/or to use another RSM.

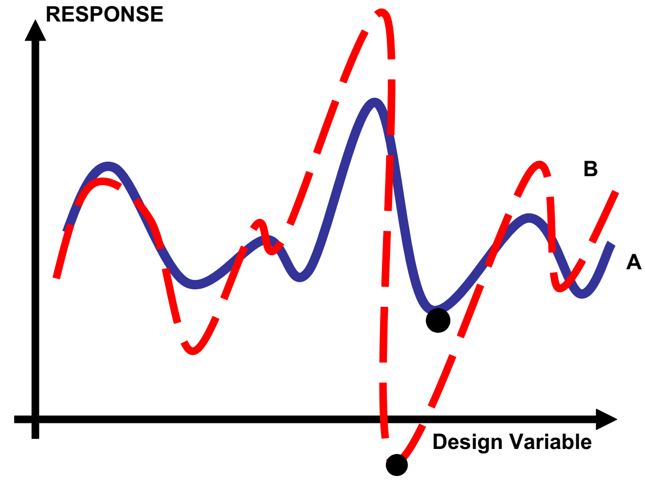

Fig. 12. Response curve for a hypothetical problem in which the objective function depends on 1 design variable.

6.3. Numerical optimisation techniques

The numerical optimisation techniques are finally used when the function is built, using a specific RSM, based on the design points suggested by the DoE technique.

There have been many optimisation algorithms proposed in the literature [31], with each having its advantages and disadvantages [29]. There are generally 2 categories of algorithms. One category is the classical gradient-based methods (i.e., its algorithm is based on the gradient of the objective function). Another is the stochastic-based methods (i.e., non gradient-based methods). Thus, essentially, the difference between the numerical optimisation techniques is based on, first of all, if their algorithm is developed or not from the gradient of the objective function, and secondly, how their algorithm works (i.e., how the searching of the local and/or global maxima and/or minima is going to be proceeded along the response surface).

In general, the behaviour of the objective function is unknown *a prior* [29]. There may exist several local minima (or maxima) in the objective function over a specific design space. The employed optimisation algorithm should be robust and possess the capability to find the global minimum (or maximum) [29–31]. Fig. 13 shows this process.

In this study, for all the problems, one gradient-based techniques and stochastic techniques were used as mentioned in the previous section. The Fletcher–Reeves and the PSO produced good results.

The Fletcher–Reeves technique is a gradient-based technique. This method is well known and can be used for constrained and unconstrained problems [16]. Its algorithm is based on the information on the first derivatives of the objective function and is considered to be very robust. The technique does not use information obtained from matrix operations which also makes it numerically efficient. The main advantages of this method are: the gradient is linearly independent of all previous direction vectors, the searching process makes good progress because it is based on gradients, the formula to determine the new direction is simple and can be used for large non-linear optimisation problems. For the Fletcher–Reeves technique, we have the main algorithm:

$$x^{k+1} = x^k \pm \alpha^k \nabla F^k$$

where x^k, x^{k+1} values of the design variables in the k and $k+1$ interaction; $F(x)$ objective function to be minimized (or maximized); ∇F gradients of the objective function, constituting the

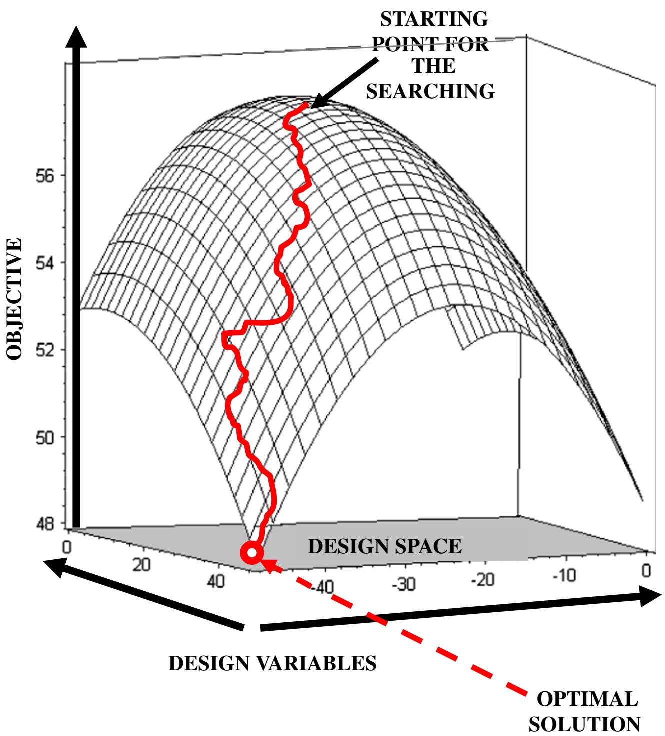

Fig. 13. Response surface for a hypothetical problem in which the objective function depends on 2 design variables.

direction of the searching; α the size of the step in the direction of the searching.

The gradient direction is defined by the following equation: $d_k = S^{(k)} = -\tilde{N}F(\underline{x})^{(k)}$
So, the new conjugate direction $(dk+1)$ is calculated according to:

$$dk+1 = -gk+1+\beta k \cdot dk$$

and βk based on the Fletcher–Reeves method is given by:

$$\beta_k = \frac{(g_{k+1})^T(g_{k+1})}{(g_k)^T(g_k)}$$

So, this algorithm defined by the Fletcher–Reeves method works through 3 main steps:

- 1) Starting at any $x0$ define $d0 = -g0$, where g is the column vector of gradients of the objective function at point $f(x)$;
- 2) Using dk, find the new point $xk+1 = xk+ak \cdot dk$, where ak is found using a line search that minimizes $f(xk+\alpha k \cdot dk)$;
- 3) Calculate the new conjugate gradient direction dk+1 , according to: $dk+1 = -gk+1+\beta k \cdot dk$.

In the other hand, non gradient-based numerical optimisation techniques are based on stochastic algorithms. The Particle Swarm Optimisation (PSO), which is a world widely used stochastic method (also called as non gradient-based method), is based on a simplified social model that is closely tied to swarming theory [16]. It was developed by Dr. Eberhart and Dr. Kennedy in 1995 and it is inspired by social behaviour of bird flocking or fish schooling [31]. The algorithm of the PSO technique is very robust. The principle is that the design variables are understood as particles with associated velocities. The method is analogue to the Fletcher–Reeves, however instead of using the gradients of the objective function to insert the searching algorithm, the vectors are represented by uniform random numbers between 0 and 1.

In the next section, the methodology here proposed is applied to hypothetical study case.

7. Application of the methodology

7.1. The scenario

The geometry is a regular and symmetrical shaped room – 10 m × 10 m square room) with one single exit.

The geometry is populated by 200 occupants and it was assumed that the response times was zero. It was also assumed that occupants would move to their nearest exits. These measures were taken in order to simplify the analyses, once this work is concerned on the relation occupants–structure.

Given the simplicity of the geometry, the domain of the design variable exit location (EL) is

$0 < \text{EL} < \text{middle of the wall}$

As explained previously, for instance, when EL = 2.5, this means that the exit is located 2.5 m far from the corner. When, EL = 0, this means that the exit is located in the corner, see Fig. 14.

The exit width (EW) was also taken into consideration. As mentioned before, this problem also considered the EW varying from 1.0 m to 2.5 m:

$1 < \text{EW} < 2.5$

Therefore, the problem to be investigated in this study, taking the exit(s) locations and the exit widths as the design variables, can be stated very simply as follows: for an enclosure of given size and shape, containing an arbitrarily large population, is there an optimal location for the exit(s) combined with the exit width that will minimize the evacuation times?

This is the question which this study tried to answer through the use of the concepts within the optimisation theory combined with the use of evacuation modelling analysis.

The simulations were performed 600 times for each scenario (i.e., design point). A total of 12,000 simulations were performed.

The optimal solution is understood as being the global minima region of the objective function (i.e., the evacuation time) in the response surface. However, given that to find precisely the global minima region is not an easy task, if the local minima region near to

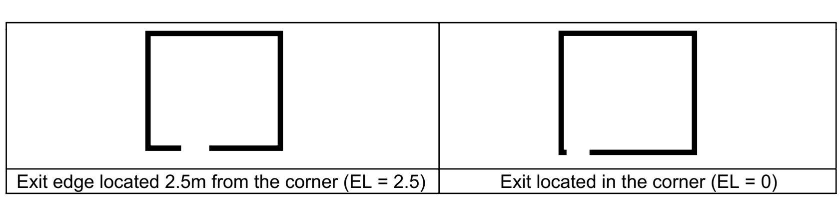

Fig. 14. Exit locations for single exit cases with 1.0 m wide door.

Table 2
Optimal solution for the squared room with one exit.

Optimum = (0; 2.5; 73.14); Response surface model; Full-quadratic (R^2 = 0.99)	Numerical optimisation technique	
DoE techniques	Fletcher–Reeves	PSO – Particle swarm optimisation
Central composite design (CCD) – 7 points	(0; 2.48; 74)	(0.5; 2.48; 74)
Central composite design (CCD) – 9 points	(0; 2.49; 73.5)	(0; 2.49; 73.5)
Latin hypercube – 6 points	(0; 2.48; 74.2)	(0.3; 2.48; 74.2)

the global minima region is found, it was also assumed that this is also a good result.

Therefore, the objective function (i.e., the evacuation time, ET) is a function with 2 variables. The statement for this problem is:

$$\text{Minimize}: \ \text{ET} = f(\text{EL}; \text{EW})$$

where ET – the evacuation time; EL – the exit locations; EW – the exit widths.

7.2. Results

Table 2 presents the results from this optimisation analysis and Fig. 15 shows the response surface model for this problem.

From previous studies [25], it was found that to have an exit in the corner for square rooms is the best location.

For this particular case, where the exit width is also varying, it was found through data analysis that the best solution is found when the exit is located in the corner (i.e., EL = 0) and the exit width is 2.5 m, producing an evacuation time of 73.14 s. This can be seen in Fig. 15.

Now using the methodology here proposed, it can be seen from the results shown on Table 2 that not the global minima region was found, nevertheless a near local minima region was found. This is a good result as well, once the coordinates also suggested that the best place to locate the exit is in the corner (EL = 0), and regarding the exit width, close values to the upper value was found (i.e., 2.48; 2.49).

For both DoE techniques and numerical optimisation techniques used, similar results were found and they are also satisfactory results, once the local minima region was found and very close to the global minima region.

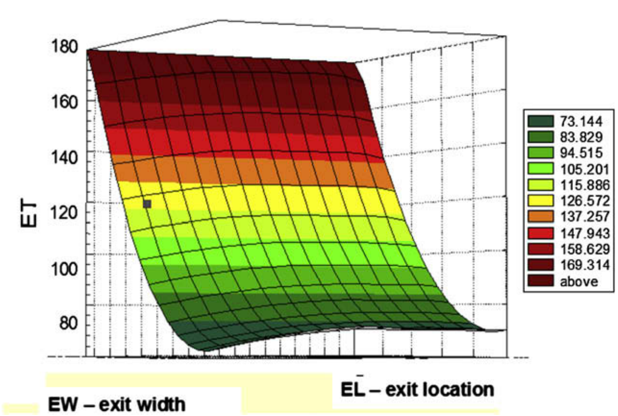

Fig. 15. Response surface for the one single exit case.

8. Concluding comments

In this paper, we have shown a combined approach which uses evacuation modelling analysis and operational research concepts for improving enclosure designs in terms of fire safety. The operational research concepts include numerical optimisation techniques and associated techniques, such as design of experiments (DoE) and response surface modelling (RSM).

For this purpose, we have explored the optimal positioning of exits around the perimeter of a square room in order to minimize the evacuation times. In reality, the exits positioning in enclosures is an important issue itself within the fire safety engineering field.

Furthermore, we have considered a simple case study from previous studies, namely a square room with one single exit, given that the best solutions were already found through data analysis. We found these solutions applying the concept of combining the use of numerical optimisation techniques with evacuation simulation modelling.

The results obtained have shown to be satisfactory, i.e., global minima and local minima closest to the global minima region were found. In other terms, previously, it was found that to have an exit of 2.5 m located in the corner of the wall would produce the lowest evacuation time (i.e., 73.14 s). Therefore, this is the global minimum solution. And now, using the approach proposed in this paper, we have found solutions within the local minima region close to the global minima region. For instance, for all the solutions, the results have shown that to have the exit located in the corner and/or close to the corner will produce minimum evacuation times (i.e., 73.5–74.2 s). And besides that, the exit width suggested by the results has also assumed similar values to 2.5 m (i.e., 2.48–2.49).

For all the cases, we have used a gradient-based algorithm (i.e., the Fletcher–Reeves numerical optimisation technique) and non gradient-based algorithm (i.e., the Particle Swarm Optimisation numerical optimisation technique) to find the optimal solution.

For this study, we defined the problem as being unconstrained.

We have used two different DoE techniques: the CCD and the Latin Hypercube techniques. Using the CCD, we have defined two sets of design points: one set using 7 design points and the other set using 9 design points. And using the Latin Hypercube, we have defined one set of 6 design points. We have found similar results.

For this problem, the response surface model based on full-quadratic seem to be appropriate, producing the regression coefficient R^2 of 0.99. The general stepwise regression was the multivariable regression analysis method selected to build these response surface models and it worked well, once the coefficients R^2 were higher than 0.90.

Both the Fletcher–Reeves and PSO methods were found to produce acceptable solutions for this problem. This approach suggests that minimum evacuation times can be achieved by positioning the exit exactly in the corner of the room. This result correctly matches that found using the "brute force method" (i.e., the results found based on data analysis).

The analysis revealed that this methodology seems to be a very powerful tool for evacuation modelling analysis.

This systematic methodology to efficiently optimise evacuation safety aspects of structural designs has been extended to more complex designs.

The author has also successfully applied the technique to rooms with two exits (this is a constrained problem as the location of the exits cannot overlap), non-square shaped rooms (i.e., such as rectangular rooms, circular and other more complex geometries, taking into account the influence of the lay-out) and rooms with exits of different sizes and the approach appears to be able to identify reasonable solutions to these problems. Further testing of the method continues to determine its robustness.

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